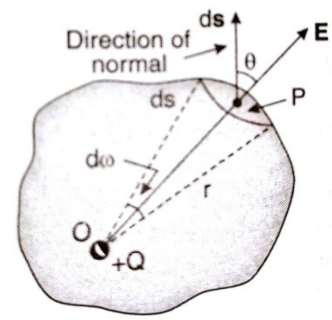
Electrostatics
(Electric field intensity and Electric Potential)
Gauss law

Statement:- The total normal electric flux passing through a closed surface is equal to $\frac{1}{\epsilon_O}$ times to the total charge contained with in the closed surface.



$$\emptyset = \int \bar{E}\,.\overline{dS} = \frac{1}{\epsilon_o}(q)$$

Case 1 :- When the charge is inside the closed surface

- "+q" is the charge located at the point O. A closed surface is considered around the charge. dS is the small area on the surface.
- dS small area is at a distance 'r' from the point 'O'.
- dS area is making'dω' solid angle at the point 'O'. Let 'θ' be the angle between $\overline{dS}$ and $\bar{E}$

$\overline{dS}$ = Areal vector.

$\bar{E}$ = Electric field intensity vector.

Electric flux passing through the area dS $\quad d\emptyset = \bar{E}\,.\overline{dS}$ = E dS Cosθ

But $\quad E = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}$

Substituting this value in the above equation $\quad d\emptyset = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}$ dS Cosθ

$$d\emptyset = \frac{q}{4\pi\epsilon_o}\frac{dS\cos\theta}{r^2}$$

$$d\emptyset = \frac{q}{4\pi\epsilon_o}d\omega \qquad \because \frac{dS\cos\theta}{r^2} = d\omega$$

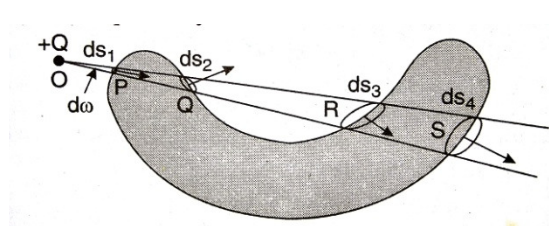
Electric flux passing through the total area $\quad \emptyset = \int d\emptyset = \int \frac{q}{4\pi\epsilon_o}d\omega$

$$\emptyset = \frac{q}{4\pi\epsilon_o}\int_0^{4\pi} d\omega = \frac{q}{4\pi\epsilon_o}[\omega]_0^{4\pi}$$

$$\emptyset = \frac{q}{\cancel{4\pi}\epsilon_o}\cancel{4\pi}$$

$$\boxed{\therefore \emptyset = \int \bar{E}\,.\overline{dS} = \frac{1}{\epsilon_o}(q)}$$

Case 2 :- When the charge is inside the closed surface



- ✓ The closed surface is as shown in the figure. +q charge is out side the closed surface and located at point 'O'.
- ✓ Take a solid angle $d\omega$ at the point 'O'.
- ✓ Let this solid angle $d\omega$ cut, the 1^{st}, 2^{nd}, 3^{rd} and 4^{th} surfaces with areas dS_1, dS_2, dS_3 and dS_4 respectively.

✓ dS_1, dS_2, dS_3 and dS_4 areal vectors are making $\theta_1, \theta_2, \theta_3, \theta_4$ angles respectively with the electric field vector $\bar{E}$.

✓ These 4 surfaces are at distances r_1, r_2, r_3, r_4 respectively from the charge +q.

$$d\emptyset = \frac{q}{4\pi\epsilon_o}\frac{dS\ Cos\theta}{r^2}$$ Applying this equation to the above 4 surfaces.

$$d\emptyset = \frac{q}{4\pi\epsilon_o}\left[\frac{dS_1\ Cos\theta_1}{{r_1}^2} + \frac{dS_2\ Cos\theta_2}{{r_2}^2} + \frac{dS_3\ Cos\theta_3}{{r_3}^2} + \frac{dS_4\ Cos\theta_4}{{r_4}^2}\right]$$

$$d\emptyset = \frac{q}{4\pi\epsilon_o}\left[- d\omega + d\omega - d\omega + d\omega \right] = 0$$

$$\boxed{\therefore \emptyset = \int \bar{E}\,.\,\overline{dS} = 0}$$

So, when the charge is out side the closed surface, the total normal electric flux is zero.

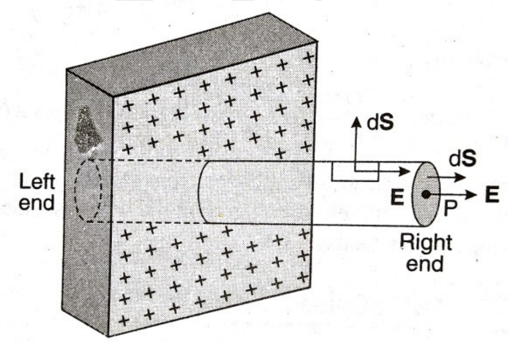
Electric field due to infinite conducting sheet of charge

❖ If charge is given to a conducting sheet of infinite area, that charge is confined to the surface area.

❖ Let 'P' be the point where the electric field is to be measured and 'σ' be the surface charge density. Then

❖ Then $\sigma = \frac{q}{A}$ (లేదా) $q = \sigma A \longrightarrow (1)$

q = Total charge give to the sheet
A = Total area of the sheet



❖ Consider a Gaussian surface in cylindrical shape perpendicular to the surface.

❖ Total electric flux passing through the Gaussian surface

$$\emptyset = \int_{Left\ end}\bar{E}\,.\,\overline{dS} + \int_{Right\ end}\bar{E}\,.\,\overline{dS} + \int_{Curved\ surface}\bar{E}\,.\,\overline{dS}$$

❖ There is no charge on the left side of the sheet and the electric field is zero on that side. So, the first integral value is zero.

❖ Regarding the curved surface, the angle between E and dS is 90^o. So, the third integral value is also zero.

$$\emptyset = 0 + \int_{Right\ end} E\ dS + 0 = E\int dS = EA$$

As per Gauss law $\emptyset = EA = \frac{q}{\epsilon_o}$

Substituting eqn.(1) $E\cancel{A} = \frac{\sigma \cancel{A}}{\epsilon_o}$ (OR) $\boxed{E = \frac{\sigma}{\epsilon_o}}$

<u>Note</u> :- If the charged sheet is made with non-conducting material, then the charge is on bothe surfaces.

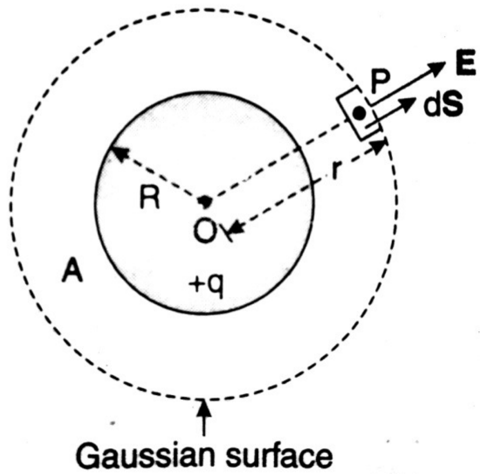
Then $\emptyset = \int_{Left\ end} E\ dS + \int_{Right\ end} E\ dS + 0 = EA + EA = 2\,EA$

$\emptyset = 2EA = \frac{q}{\epsilon_o} = \frac{\sigma A}{\epsilon_o}$ (OR) $\boxed{E = \frac{\sigma}{2\epsilon_o}}$

Electric field due to uniformly charge (Non-conducting) Sphere

- Let +q be the charge given to the non-conducting solid sphere of radius 'R' having the centre at the point 'O'. Then the charge is distributed uniformly through out the sphere.

Case 1 :- At a point out side the sphere



- Let 'P' be the point where the electric field intensity (E) is to be measured. This point 'P' is at a distance 'r' from the centre of the sphere.
- Then, imagine a Gaussian surface passing through the point 'P'.
- Consider a small area 'dS' on the Gaussian surface. Its areal vector is in the direction of $\overline{dS}$. The electric field vector $\bar{E}$ is also in the direction of $\overline{dS}$.
- As per Gauss law $\int \bar{E}\,.\overline{dS} = \frac{1}{\epsilon_o}(q)$

$$\int E\, dS\, Cos\,\theta = \frac{q}{\epsilon_o}$$

θ = The angle between $\bar{E}$ and $\overline{dS}$ = 0°

$$E \int_0^{4\pi r^2} dS = \frac{q}{\epsilon_o}$$

$$E\,[S]_0^{4\pi r^2} = \frac{q}{\epsilon_o} \quad \text{(OR)} \quad E\;4\pi r^2 = \frac{q}{\epsilon_o}$$

$$\therefore\; E = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2} \quad \text{(OR)} \quad E \propto \frac{1}{r^2} \quad \text{(OR)} \quad \frac{E_1}{E_2} = \frac{r_2^2}{r_1^2}$$

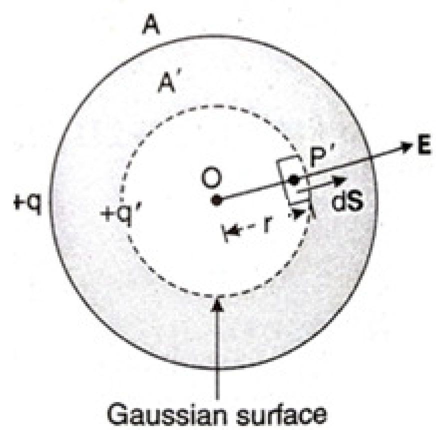
Case 2 :- Point on the surface the sphere

If the point is taken on the surface of the sphere.
Then, r = R = Radius of the sphere
substituting this value in the above equation.

$$\therefore\; E = \frac{1}{4\pi\epsilon_o}\frac{q}{R^2}$$

This has the maximum. value.

Case 3 :- At a point in side the sphere



- Let 'P¹' be the point in side the sphere where the electric field intensity (E) is to be measured. This point 'P¹' is at a distance 'r' from the centre of the sphere.
- Then, imagine a Gaussian surface passing through the point 'P¹'.
- Consider a small area 'dS' on the Gaussian surface. Its areal vector is in the direction of $\overline{dS}$. The electric field vector $\bar{E}$ is also in the direction of $\overline{dS}$.

As per Gauss law $\int \bar{E}\,.\overline{dS} = \frac{1}{\epsilon_o}(q^1) \longrightarrow (1)$

q^1 = Charge with in the Gaussian surface

Charge density $\rho = \frac{q}{V} = \frac{q}{\frac{4}{3}\pi R^3}$

If ' V^1' is the volume with in the Gaussian surface, then

$$V^1 = \frac{4}{3}\pi r^3$$

$$q^1 = \rho V^1 = \frac{q}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3$$

$$q^1 = q\frac{r^3}{R^3} \longrightarrow (2)$$

Substituting eqn.(2) in eqn. (1)

$$\int \bar{E}.\overline{dS} = \frac{1}{\epsilon_o}\left(q\frac{r^3}{R^3}\right)$$

$$\int E\, dS\, Cos\,\theta = \frac{1}{\epsilon_o}\left(q\frac{r^3}{R^3}\right)$$

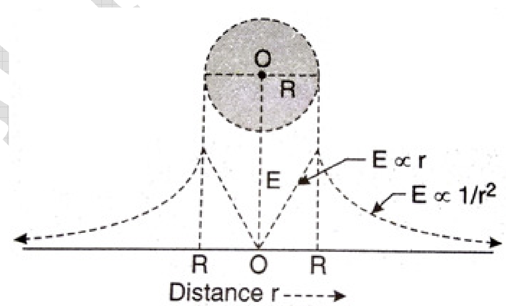
θ = The angle between $\bar{E}$ and $\overline{dS}$ = 0^o

$$E\int_0^{4\pi r^2} dS = \frac{1}{\epsilon_o}\left(q\frac{r^3}{R^3}\right)$$

$$E\,[S]_0^{4\pi r^2} = \frac{1}{\epsilon_o}\left(q\frac{r^3}{R^3}\right) \quad \text{(OR)} \quad E\,4\pi r^2 = \frac{1}{\epsilon_o}\left(q\frac{r^3}{R^3}\right)$$

$$\boxed{\therefore\; E = \frac{q}{4\pi\epsilon_o}\frac{r}{R^3}} \quad \text{(OR)} \quad \boxed{E \propto r} \quad \text{(OR)} \quad \boxed{\frac{E_1}{E_2} = \frac{r_1}{r_2}}$$

Graph :- The graph drawn by taking the electric field intensity on Y – axis, the distance between the centre of the sphere and the point where the electric field intensity is to be measure on X – axis is as shown in the figure.



- ☞ Electric field intensity is directly proportional to the distance from the centre, with in the sphere. $E \propto r$
- ☞ Electric field intensity has the maximum value on the surface of the sphere.
- ☞ Electric field intensity is inversly proportional to the square of the distance from the centre, out side the sphere. $E \propto \frac{1}{r^2}$

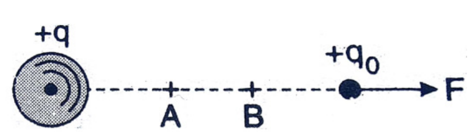
Electric Potential

Definition :- The electric potential at a point in the electric field is the work done in moving the unit positive charge from infinite distance to that point.

Potential Difference:- Potential difference between two points in an electric field is the work done in moving the unit positive charge from one point to the other point.

Explanation :-

- ✓ Consider the electric field around '+q' charge.
- ✓ Let W_{AB} is the work done in moving a test charge q_o from the Point B to A in the electric field.



Work done in moving the charge $q_o = W_{AB}$

Work done in moving the unit positive charge = $\frac{W_{AB}}{q_o}$

This is the potential difference (V_{AB}) between the points A and B.

$$V_{AB} = \frac{W_{AB}}{q_o} \quad \text{(OR)} \quad V = \frac{W}{q}$$

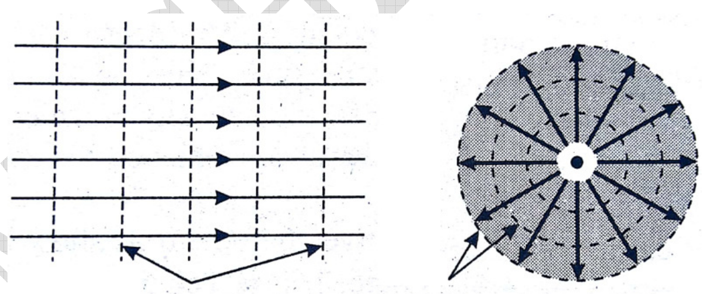
- ✓ Electric potential is measured in “Volts”.
- ✓ As per the above equation $1\ V = \frac{1\ J}{1\ C}$

Volt definition :- If one joule is the work done in moving the one coulomb charge from one point to the other in an electric field, then the potential difference between those two points is called one Volt.

Equi-potential Surfaces :-

Consider the electric field around a charge “+q”. Consider a plane that joins all the points having the same potential. This plane is called equi-potential surface.

Definition :- Equi-potential surface is the locus of all points in the electric field which have the same electric potential.



(a) Equipotential Surfaces (b)

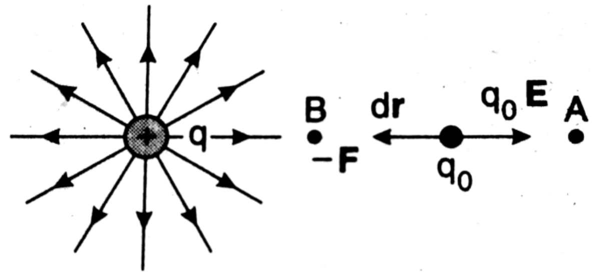
- ❖ If the charge is in point size then the equi-potential surfaces are concentric spheres.
- ❖ Equi-potential surfaces are perpendicular to the electric field vector.
- ❖ In uniform electric field the equi-potential surfaces are parallel planes.

Properties :-

1. Definition :- Equi-potential surface is the locus of all points in the electric field which have the same electric potential.
2. If the charge is in point size then the equi-potential surfaces are concentric spheres.
3. In uniform electric field the equi-potential surfaces are parallel planes.
4. Equi-potential surfaces are perpendicular to the electric field vector.
5. The work done in moving a unit positive charge from one point to the other on the same equi-potential surface is zero. i.e. The potential difference between any two points on the same equi-potential surface is zero.
6. The component of electric field vector on the equi-potential surface is zero.
7. These equi-potential surfaces resembles the wave fronts in optics. Similarly, Electric field vectors resembles the light rays or light waves.

Electric Potential due to a point charge

- Let "B" be the point in the electric field of the charge "q" where the electric field is to be measured.
- Let q_o be the test charge placed at a distance 'x' from the charge q.
- The electrostatic force between q and q_o $F = \frac{1}{4\pi\epsilon_o}\frac{qq_o}{x^2}$
- If both q and qo are positive charges. THen the repulsive force act between them.



Then $F = -\frac{1}{4\pi\epsilon_o}\frac{qq_o}{x^2}$

Work to be done to move q_o charge through a distance dx

$$dW = F.dx = -\frac{1}{4\pi\epsilon_o}\frac{qq_o}{x^2}.dx$$

Work to be done to move charge q_o from infinite distance to a point B which is at a distance r from the charge q $W = \int_{\infty}^{r} dW = \int_{\infty}^{r} -\frac{1}{4\pi\epsilon_o}\frac{qq_o}{x^2}.dx$

$$W = -\frac{qq_o}{4\pi\epsilon_o}\left[\frac{-1}{x}\right]_{\infty}^{r} = \frac{qq_o}{4\pi\epsilon_o}\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

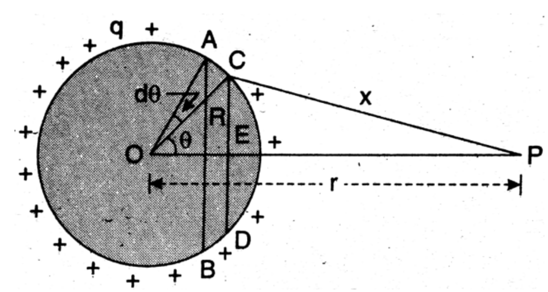
$$W = \frac{1}{4\pi\epsilon_o}\frac{qq_o}{r}$$

Work done to move unit positive charge from infinite distance to the point B is the potential at the point B. So, potential $V = \frac{W}{q_o} = \frac{1}{4\pi\epsilon_o}\frac{q}{r}$

$$\therefore V = \frac{1}{4\pi\epsilon_o}\frac{q}{r}$$

Electric potential due to charged spherical shell (Hallow sphere)

- Consider a hallow spherical conductor having radius 'R' and centre at 'O'
- If a charge 'q' is given to the spherical shell, the charge is uniformly distributed on the surface of the shell.
- Let P be the point where the potential is to be measured and it is at a distance 'r' from the centre of the sphere.



Surface charge density $\sigma = \frac{q}{4\pi R^2}$ ⟶ (1)

- Consider a small element ABCD (circular ring) on the sphere.
- Let the angle between OP, OC is θ. Let the angle between OC,OA is dθ.

The radius of the circular ring ABCD = $R\ Sin\theta$

Circumferance of the circular ring = $2\,\pi\,R\,Sin\theta$
width of the circular ring = $R.d\theta$
Surface area of the circular ring = $2\,\pi\,R\,Sin\theta\;(R.d\theta) = 2\,\pi\,R^2\,Sin\theta\;d\theta$
Charge on the surface of the circular ring $dq = 2\,\pi\,R^2\,Sin\theta\;d\theta.\sigma \longrightarrow$ (2)
Substituting eqn. (1) in eqn.(2)

$$dq = 2\,\pi\,R^2\,Sin\theta\;d\theta.\frac{q}{4\pi R^2}$$

$$dq = \frac{q}{2}.Sin\theta\;d\theta \longrightarrow (3)$$

The edge of the circular ring is at a distance 'X' from the point P.
The potential at the point P due to the charge on the circular ring

$$dV = \frac{1}{4\pi\epsilon_o}\frac{dq}{x} \longrightarrow (4)$$

Substituting eqn (3) in eqn.(4)

$$dV = \frac{1}{4\pi\epsilon_o x}.\frac{q}{2}.Sin\theta\;d\theta$$

$$dV = \frac{1}{8\pi\epsilon_o}.\frac{qSin\theta\;d\theta}{x} \longrightarrow (5)$$

From the Δ^{le} OCP $\quad x^2 = R^2 + r^2 - 2Rr\,Cos\theta$

X and θ are variables in the above eqn. because their values vary with the position of the circular ring
Differentiating the above eqn. $\cancel{2}\,x\,dx = 0 + 0 - \cancel{2}Rr\,(-Sin\theta.d\theta)$

$$x\,dx = Rr\,(Sin\theta.d\theta)$$

$$\therefore\; Sin\theta.d\theta = \frac{x.dx}{Rr} \longrightarrow (6)$$

Substituting eqn (6) in eqn.(5)

$$dV = \frac{1}{8\pi\epsilon_o}.\frac{q}{x}.\frac{x.dx}{Rr}$$

$$\therefore\; \boxed{dV = \frac{q}{8\pi\epsilon_o\,Rr}\;dx} \longrightarrow (7)$$

Case 1 : - Potential out side the sphere

If the point P lies out side the sphere, then the integration limits are from (r-R) to (r+R).

The potential due to the total charge on the sphere $V = \int_{(r-R)}^{(r+R)} dV = \int_{(r-R)}^{(r+R)} \frac{q}{8\pi\epsilon_o\,Rr}\;dx$

$$V = \frac{q}{8\pi\epsilon_o\,Rr}\int_{(r-R)}^{(r+R)} dx$$

$$V = \frac{q}{8\pi\epsilon_o\,Rr}\;(x)_{(r-R)}^{(r+R)}$$

$$V = \frac{q}{8\pi\epsilon_o\,Rr}\;(\cancel{r} + R - \cancel{r} + R)$$

$$V = \frac{q}{8\pi\epsilon_o\,\cancel{R}r}.2\cancel{R}$$

$$V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r}$$

(OR) $V \propto \frac{1}{r}$ (OR) $\frac{V_1}{V_2} = \frac{r_2}{r_1}$

Case 2 :- Potential on the surface of the sphere

When the point P lies on the surface of the sphere, then r = R

$$\therefore V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{R}$$

Case 3 : - Potential in side the sphere

When the point P lies on the surface of the sphere, then the integration limits are from (R+r) to (R-r)

Then the potential $V = \int_{(R-r)}^{(R+r)} dV = \int_{(R-r)}^{(R+r)} \frac{q}{8\pi\epsilon_0 Rr} dx$

$$V = \frac{q}{8\pi\epsilon_0 Rr}\int_{(R-r)}^{(R+r)} dx$$

$$V = \frac{q}{8\pi\epsilon_0 Rr}(x)_{(R-r)}^{(R+r)}$$

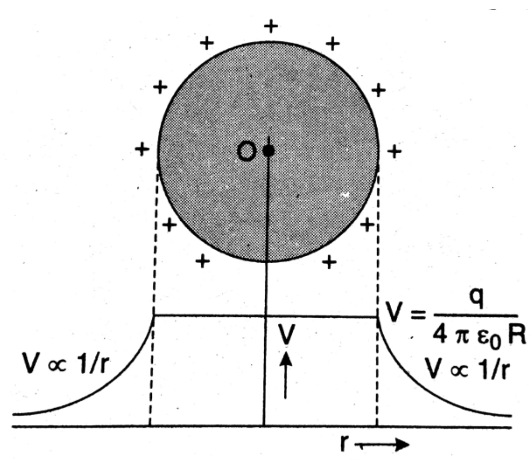
$$V = \frac{q}{8\pi\epsilon_0 Rr}(\not{R} + r - \not{R} + r)$$

$$V = \frac{q}{8\pi\epsilon_0 R\not{r}}\cdot 2\not{r}$$

$$V = \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{R}$$

So, the potential in side the sphere is same at all points and its value is equal to that on the surface of the sphere.

Graph :- The potential in side the sphere is same at all points. So, the graph is parallel straight line to the distance axis. And out side the sphere the V value decreases with the increasing value of 'r' $\left(V \propto \frac{1}{r}\right)$.



Courtesy:

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